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II. Solution by S. A. COREY, Hiteman, Iowa.

We have $1/y = \int_0^\infty e^{-ay} da$ if $y > 0$. Hence

$$\begin{aligned}\int_0^\infty \frac{\sin my}{y} dy &= \int_0^\infty \sin my \int_0^\infty e^{-ay} da dy = \int_0^\infty \int_0^\infty \sin my e^{-ay} da dy, \\ &= \int_0^\infty \int_0^\infty \sin my e^{-ay} dy da = \int_0^\infty \frac{mda}{a^2 + m^2} \\ &= \frac{1}{2}\pi \text{ if } m > 0, = -\frac{1}{2}\pi \text{ if } m < 0, = 0 \text{ if } m = 0. \\ (\text{See Byerly's } \textit{Integral Calculus}, \text{ page 100.}) \text{ Similarly,} \\ \int_0^\infty \frac{\cos my}{y} dy &= \int_0^\infty \cos my \int_0^\infty e^{-ay} da dy = \int_0^\infty \frac{ada}{a^2 + m^2} = 0,\end{aligned}$$

for all finite real values of m .

III. Remark by F. D. POSEY, A. B., San Mateo, Cal.

A discussion of the first of these integrals will be found on page 271, Art. 285 of Todhunter's *Integral Calculus*. The result there obtained is $\frac{1}{2}\pi$ if m be positive, $-\frac{1}{2}\pi$ if m be negative.

Also solved by G. W. Greenwood, G. B. M. Zerr, and J. Scheffer.

MISCELLANEOUS.

146. Proposed by F. P. MATZ, Ph. D., Sc. D.

Given $\begin{cases} a \cos \alpha + b \sin \alpha = c \\ a \cos \beta + b \sin \beta = c \end{cases}$ to prove that

$$\sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}, \text{ and } \cot \alpha + \cot \beta = \frac{2ab}{c^2 - a^2}.$$

I. Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

We have at once $\frac{\cos \alpha - \cos \beta}{\sin \alpha - \sin \beta} = -b/a$ (1).

Now $\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$ and $\sin \alpha - \sin \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \times \sin \frac{1}{2}(\alpha - \beta)$. Substituting these values in (1) we obtain $\tan \frac{1}{2}(\alpha + \beta) = b/a$. Since $\tan \frac{1}{2}(\alpha + \beta) = \pm \sqrt{\frac{1 - \cos(\alpha + \beta)}{1 + \cos(\alpha + \beta)}} = b/a$, and solving,

$$\cos(\alpha + \beta) = \frac{a^2 - b^2}{a^2 + b^2} \text{ (2); } \sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2} \text{ (3).}$$

Multiplying together the given equation and substituting the values of $\cos(\alpha + \beta)$,

$\sin(\alpha+\beta)$ from (2) and (3) we find that $\sin\alpha\sin\beta=\frac{c^2-a^2}{a^2+b^2}$; and therefore

$$\cot\alpha+\cot\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}=\frac{2ab}{c^2-a^2}.$$

II. Solution by L. S. SHIVELY, Mt. Morris College, Mt. Morris, Ill.

Subtracting the second equation from the first, $a(\cos\alpha-\cos\beta)+b(\sin\alpha-\sin\beta)=0$. Hence, $\sin\frac{1}{2}(\alpha-\beta)[a\sin\frac{1}{2}(\alpha+\beta)-b\cos\frac{1}{2}(\alpha+\beta)]=0$. Either $\sin\frac{1}{2}(\alpha-\beta)=0$ or $a\sin\frac{1}{2}(\alpha+\beta)=b\cos\frac{1}{2}(\alpha+\beta)$. In the first case we have the trivial solution $\alpha=\beta$. In the second case, we put $(\alpha+\beta)=x$, $\sin\frac{1}{2}x=b/a\cos\frac{1}{2}x$. Since $\sin x=2\sin\frac{1}{2}x\cos\frac{1}{2}x$ the latter equation takes the form $(a^2+b^2)\sin^2x=2ab\sin x$, whence, excluding the value $\sin x=0$ introduced by squaring, $\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}$.

Secondly, $\cot\alpha+\cot\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$. From the first of the given equations

$$a\sqrt{1-\sin^2\alpha}+b\sin\alpha=c. \text{ Solving for } \sin\alpha, \text{ we have } \sin\alpha=\frac{bc\pm a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}$$

$$\text{Similarly, solving the second of the given equations, } \sin\beta=\frac{bc\pm a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}$$

Since the solution $\alpha=\beta$ was rejected, $\sin\alpha\sin\beta=\frac{c^2-a^2}{a^2+b^2}$.

$$\text{Hence } \cot\alpha+\cot\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta} \text{ becomes equal to } \frac{2ab}{c^2-a^2}.$$

III. Solution by A. H. HOLMES, Brunswick, Maine.

From $a\cos\alpha+b\sin\alpha=c$,

$$\sin\alpha=\frac{bc+a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}, \quad \cos\alpha=\frac{ac-b\sqrt{(a^2+b^2-c^2)}}{a^2+b^2},$$

and from $a\cos\beta+b\sin\beta=c$,

$$\sin\beta=\frac{bc-a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}, \quad \cos\beta=\frac{ac+b\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}.$$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$. Putting for $\sin\alpha$, $\cos\alpha$, $\sin\beta$, and $\cos\beta$ their values above, and reducing, $\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}$, $\cot\alpha+\cot\beta=\frac{\cos\alpha}{\sin\alpha}+\frac{\cos\beta}{\sin\beta}$.

Introducing the values of these sines and cosines, and reducing, $\cot\alpha+\cot\beta=\frac{2ab}{c^2-a^2}$.

Also solved by G. B. M. Zerr, G. W. Greenwood, J. Scheffer, and E. L. Rich.